6. Deformation of Solids

6.2 Elastic and plastic behaviour

Paper 2

Question Paper

1 A pinball machine uses a spring to launch a small metal ball of mass 4.5×10^{-2} kg up a ramp. The spring is compressed by 8.0×10^{-2} m and held in equilibrium, as shown in Fig. 4.1.

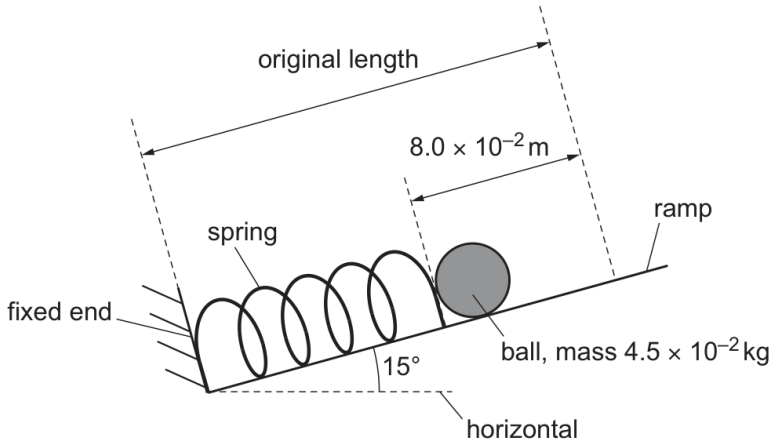


Fig. 4.1 (not to scale)

The ramp is at an angle of 15° to the horizontal.

(a) The spring obeys Hooke's law and has a spring constant of $29\,N\,m^{-1}$.

Calculate the elastic potential energy in the compressed spring.

elastic potential energy = ... J [2]

2 **(b)** A metal wire P that obeys Hooke's law is stretched within its limit of proportionality.

(iii) State the name of the quantity represented by the area under the line in Fig. 4.1.

..... [1]

3 **(a)** The variation of stress with strain for a metal P is shown in Fig. 3.1.

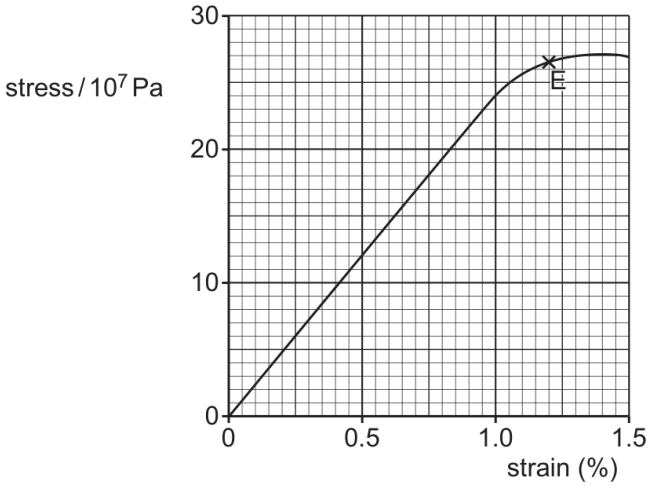


Fig. 3.1

Point E is the elastic limit of the metal.

(c) A wire is used to hold a uniform shelf AB horizontally in equilibrium as shown in Fig. 3.2.

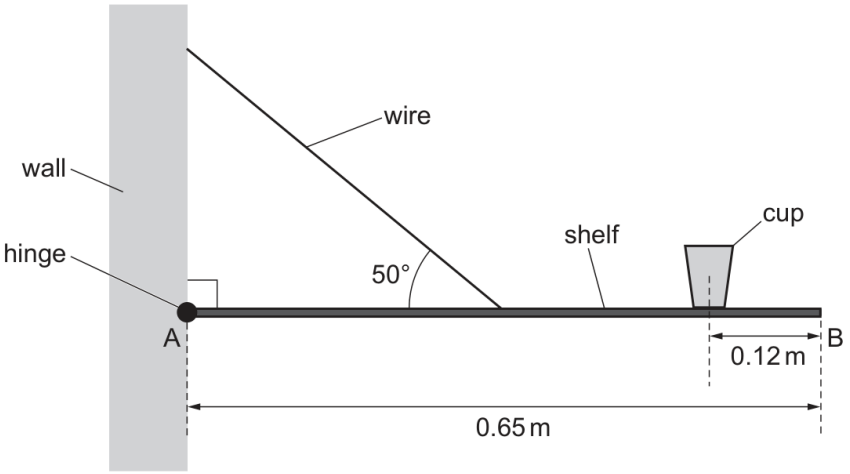


Fig. 3.2 (not to scale)

The wire is connected to the midpoint of shelf AB at an angle of 50° to the horizontal. The shelf is attached to a wall by a hinge at A. The length of shelf AB is 0.65 m and its weight is 33 N.
A cup of weight 1.5 N rests on the shelf with its centre of gravity at a horizontal distance of 0.12 m from B.

(iii) More items are added to the shelf, doubling the stress in the wire. The wire is made of the metal P from **(a)**.

Use Fig. 3.1 to state and explain whether the wire will behave plastically or elastically as the stress doubles.

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..... [2]

4 A rigid uniform beam of weight *W* is connected to a fixed support by a hinge, as shown in Fig. 2.1.

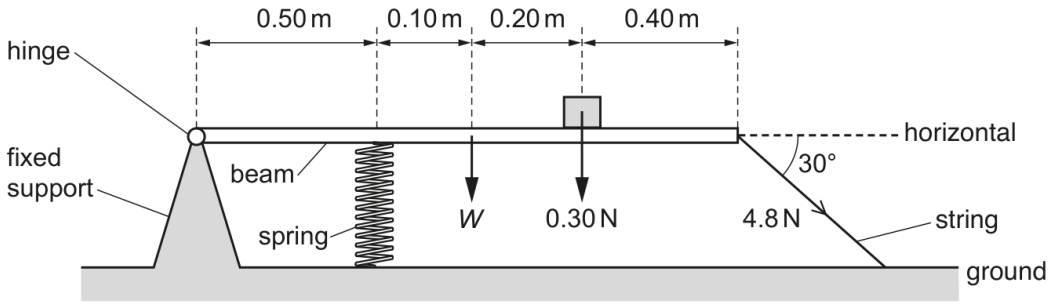


Fig. 2.1 (not to scale)

A compressed spring exerts a total force of 8.2 N vertically upwards on the horizontal beam. A block of weight 0.30 N rests on the beam. The right-hand end of the beam is connected to the ground by a string at an angle of 30° to the horizontal. The tension in the string is 4.8 N. The distances along the beam are shown in Fig. 2.1.

The beam is in equilibrium. Assume that the hinge is frictionless.

(b) The spring obeys Hooke's law and has an elastic potential energy of 0.32 J.

Calculate the compression of the spring.

compression = ... m [2]

5 A spring is suspended from a fixed point at one end. The spring is extended by a vertical force applied to the other end. The variation of the applied force F with the length L of the spring is shown in Fig. 4.1.

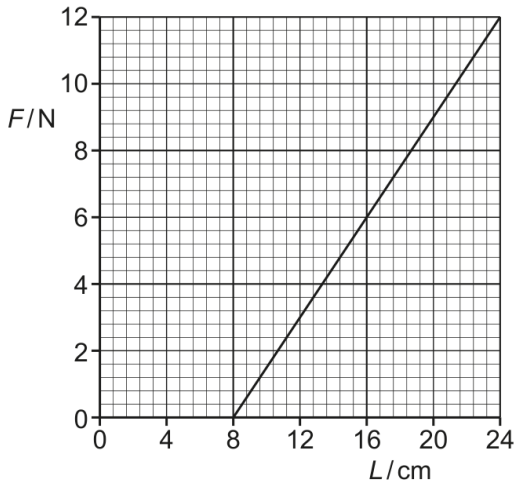


Fig. 4.1

For the spring:

(c) determine the elastic potential energy when $F = 6.0\,\text{N}$.

elastic potential energy = ... J [2]

6 A hot-air balloon floats just above the ground. The balloon is stationary and is held in place by a vertical rope, as shown in Fig. 2.1.

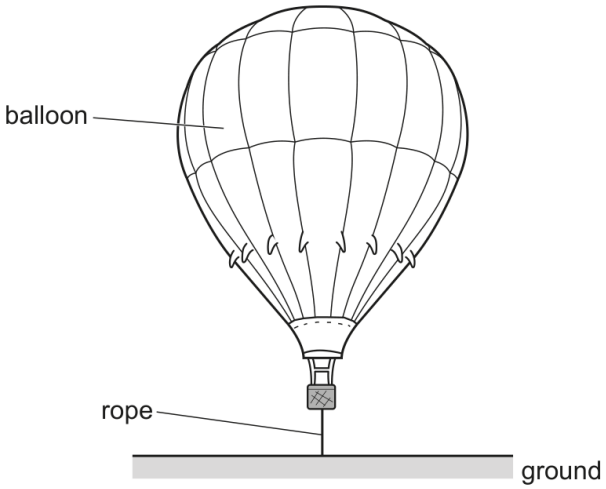


Fig. 2.1

The balloon has a weight W of 3.39×10^4 N. The tension T in the rope is 4.00×10^2 N.
Upthrust U acts on the balloon.
The density of the surrounding air is $1.23\,\text{kg}\,\text{m}^{-3}$.

(c) Before the balloon is released, the rope holding the balloon has a strain of 2.4×10^{-5}. The rope has an unstretched length of 2.5 m. The rope obeys Hooke's law.

(ii) Calculate the elastic potential energy E_P of the rope.

E_P = ... J [2]

7 A vertical rod is fixed to the horizontal surface of a table, as shown in Fig. 3.1.

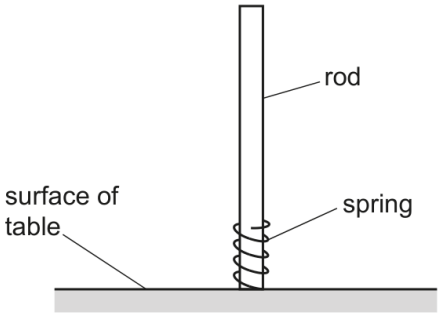


Fig. 3.1 (not to scale)

A spring of mass 7.5 g is able to slide along the full length of the rod.

The spring is first pushed against the surface of the table so that it has an initial compression of 2.1 cm. The spring is then suddenly released so that it leaves the surface of the table with a kinetic energy of 0.048 J and then moves up the rod.

Assume that the spring obeys Hooke's law and that the initial elastic potential energy of the compressed spring is equal to the kinetic energy of the spring as it leaves the surface of the table. Air resistance is negligible.

(a) By using the initial elastic potential energy of the compressed spring, calculate its spring constant.

spring constant = ... $N\,m^{-1}$ [2]

(d) The rod is replaced by another rod that exerts negligible frictional force on the moving spring. The initial compression x of the spring is now varied in order to vary the maximum increase in height Δh of the spring after leaving the surface of the table. Assume that the spring obeys Hooke's law for all compressions.

On Fig. 3.2, sketch a graph to show the variation with x of Δh. Numerical values are not required.

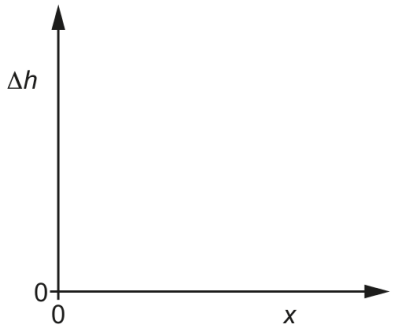


Fig. 3.2

[2]

8 A spring is suspended from a fixed point at one end and a vertical force is applied to the other end, as shown in Fig. 4.1.

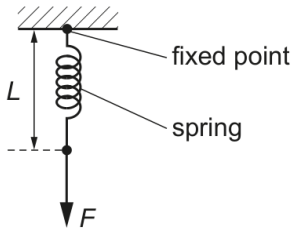


Fig. 4.1

The variation of the applied force *F* with the length *L* of the spring is shown in Fig. 4.2.

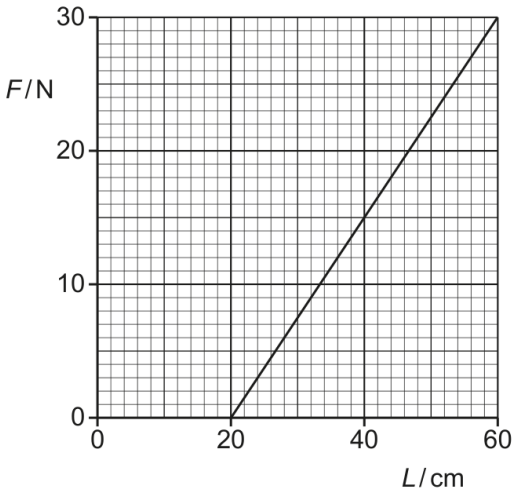


Fig. 4.2

(b) Determine the elastic potential energy in the spring when the applied force *F* is 15 N.

elastic potential energy = ... J [3]

9 A sphere is attached by a metal wire to the horizontal surface at the bottom of a river, as shown in Fig. 2.1.

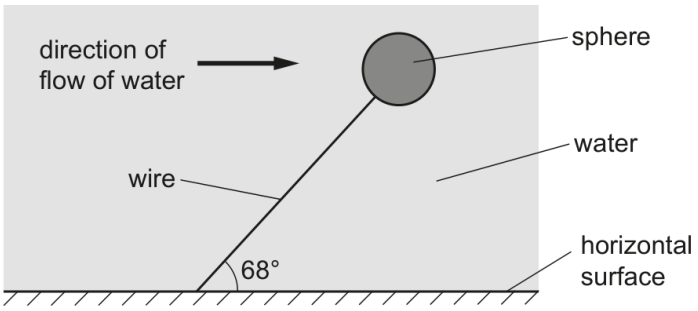


Fig. 2.1 (not to scale)

The sphere is fully submerged and in equilibrium, with the wire at an angle of 68° to the horizontal surface. The weight of the sphere is 32 N. The upthrust acting on the sphere is 280 N. The density of the water is 1.0×10^3 kg m^{-3}.

Assume that the force on the sphere due to the water flow is in a horizontal direction.

(d) The extension of the wire increases when the sphere changes position as described in **(c)**. The wire obeys Hooke's law.

(ii) Before the sphere changed position, the initial elastic potential energy of the wire was 0.65 J. The change in position of the sphere causes the extension of the wire to double.

Calculate the final elastic potential energy of the wire after the sphere has changed position.

final elastic potential energy = ... J [2]

10 A steel ball is projected horizontally from the top of a table, as shown in Fig. 2.1.

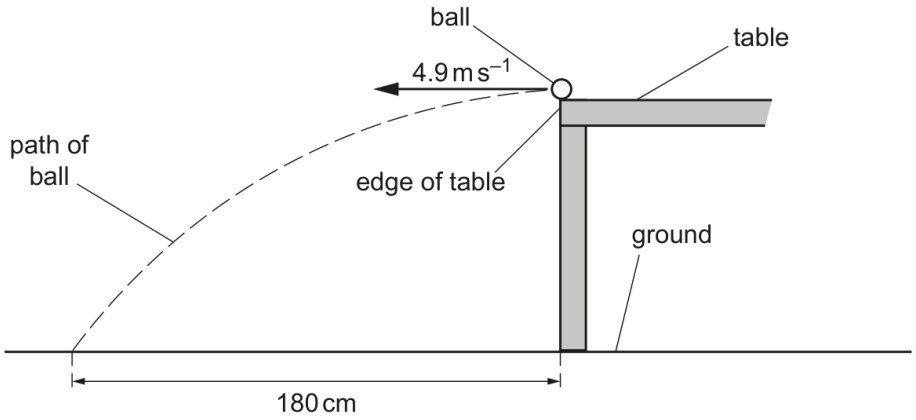


Fig. 2.1 (not to scale)

The ball is projected horizontally at a speed of 4.9 m s^{-1}. The ball lands on the ground a horizontal distance of 180 cm from the edge of the table.

Assume that air resistance is negligible.

(b) The ball is projected by means of a compressed spring which is attached to a fixed block as shown in Fig. 2.2.

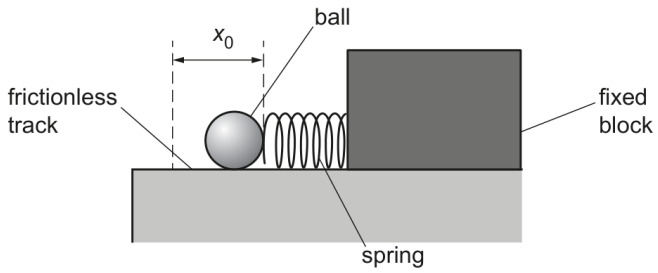


Fig. 2.2

The ball is placed on a frictionless track in front of the spring. The ball is then pulled back so that the spring has compression x_0.

When the spring is released, the ball is projected horizontally as shown in Fig. 2.3.

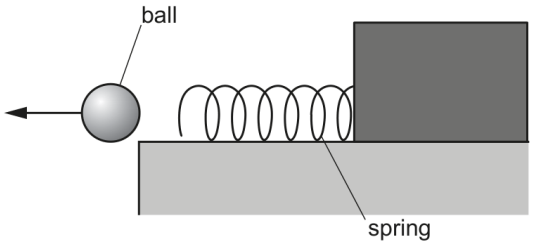


Fig. 2.3

The variation with compression x of the applied force F for the spring is shown in Fig. 2.4.

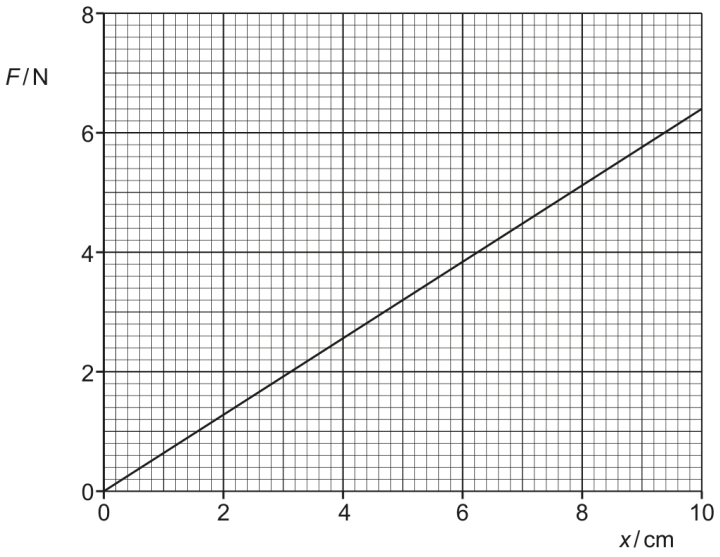


Fig. 2.4

The ball is a uniform sphere of steel of diameter 0.016 m and mass 0.017 kg.

(ii) All of the elastic potential energy in the spring is converted into kinetic energy of the ball. The speed of the ball as it leaves the spring is $4.9\,m\,s^{-1}$.

Show that the maximum elastic potential energy of the spring is 0.20 J.

(iii) Use Fig. 2.4 to determine the spring constant k of the spring.

(iv) Use your answer in **(b)(iii)** and the value of energy given in **(b)(ii)** to determine the compression x_0 of the spring.

x_0 = ... m [2]

11 A horizontal spring is fixed at one end. A block is pushed against the other end of the spring so that the spring is compressed, as shown in Fig. 4.1.

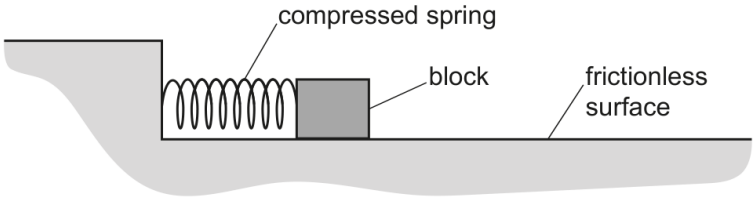


Fig. 4.1

The block is released and accelerates along a horizontal frictionless surface as the spring returns to its original length. The block leaves the end of the spring with a speed of 2.3 m s^{-1}, as shown in Fig. 4.2.

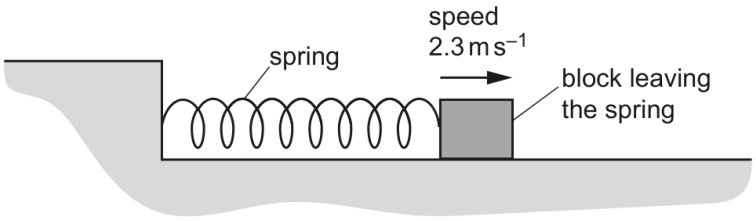


Fig. 4.2

The block has a mass of 250 g and the spring has a spring constant of 420 N m^{-1}.

Assume that the spring always obeys Hooke's law and that all the elastic potential energy of the spring is transferred to the kinetic energy of the block.

(b) Calculate the compression of the spring immediately before the block is released.

compression = ... m [2]

(d) The maximum compression x of the spring is now varied in order to vary the kinetic energy E_K of the block as it leaves the spring. Assume that all the elastic potential energy in the spring is always transferred to the kinetic energy of the block.

On Fig. 4.3, sketch a graph to show the variation with x of E_K.

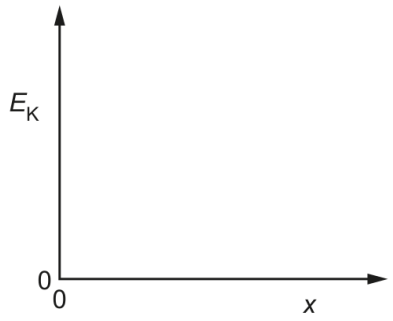


Fig. 4.3

[1]

12 (c) A floating solid cylinder is attached by a wire to the sea bed, as shown in Fig. 1.1.

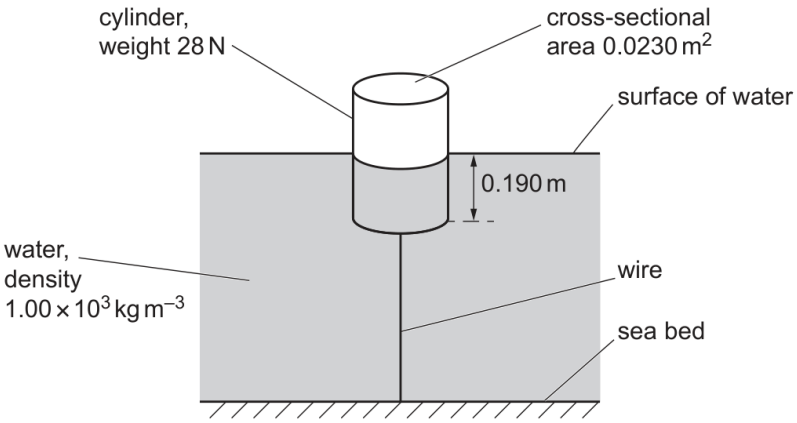


Fig. 1.1 (not to scale)

The density of the water is $1.00 \times 10^3\,\text{kg m}^{-3}$. The base of the cylinder is at a depth of 0.190 m below the surface of the water. The cylinder has a weight of 28 N and a cross-sectional area of $0.0230\,\text{m}^2$.

The wire and the central axis of the cylinder are both vertical. The cylinder is in equilibrium.

(iv) The surface of the water gradually rises until it is level with the top face of the cylinder.

State and explain, qualitatively, the variation of the strain energy stored in the wire as the water surface rises.

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..... [2]

13 **(b)** A copper wire is stretched by a tensile force that gradually increases from 0 to 280 N. The variation with extension of the tensile force is shown in Fig. 3.3.

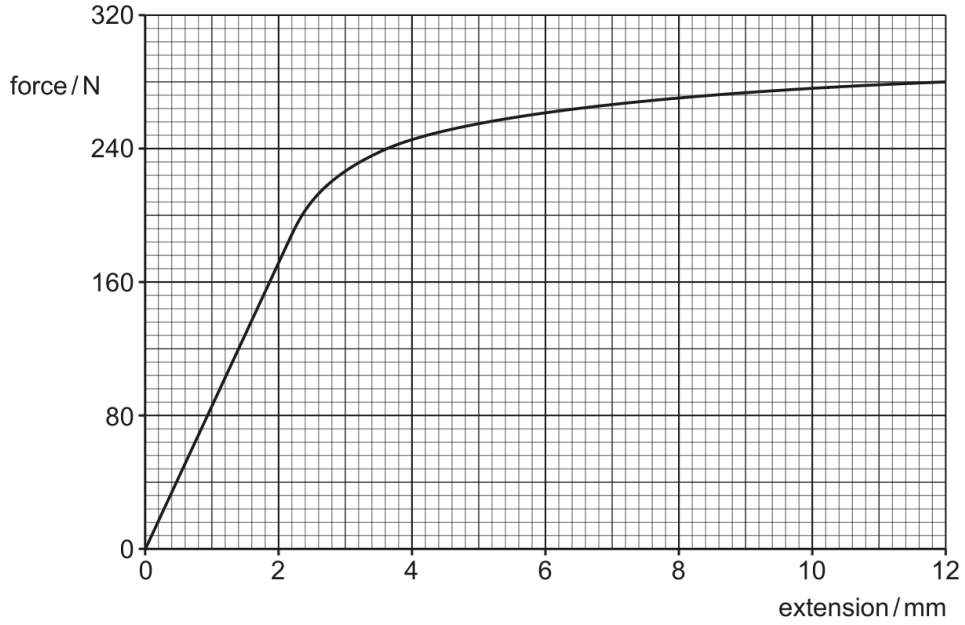


Fig. 3.3

(ii) Use Fig. 3.3 to determine the strain energy in the wire when the tensile force is 120 N.

strain energy = ... J [3]

(iii) Explain why the work done in stretching the wire to an extension of 12 mm is not equal to the energy recovered when the tensile force is removed.

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..... [2]

14 One end of a wire is attached to a fixed point. A force F is applied to the wire to cause extension x. The variation with F of x is shown in Fig. 5.1.

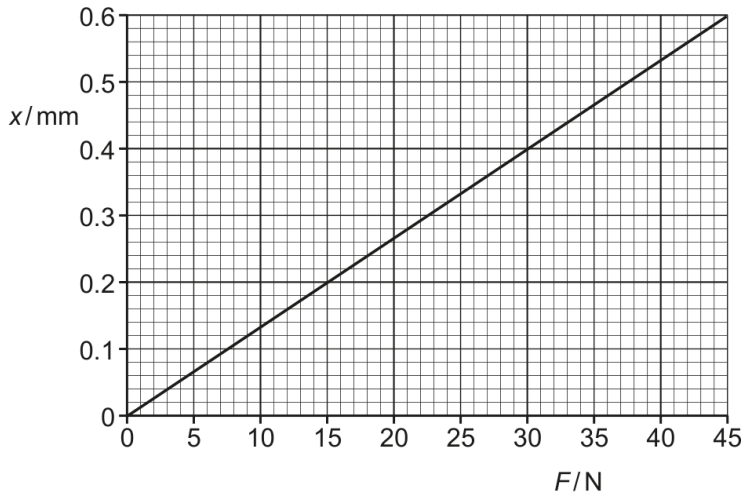


Fig. 5.1

The wire has a cross-sectional area of $4.1 \times 10^{-7}\,m^2$ and is made of metal of Young modulus $1.7 \times 10^{11}\,Pa$. Assume that the cross-sectional area of the wire remains constant as the wire extends.

(d) A force of greater than 45 N is now applied to the wire.

Describe how it may be checked that the elastic limit of the wire has not been exceeded.

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..... [1]

15 (b) (i) A school experiment is performed on a metal wire to determine the Young modulus of the metal. A force is applied to one end of the wire which is fixed at the other end. The variation of the force F with extension x of the wire is shown in Fig. 4.1.

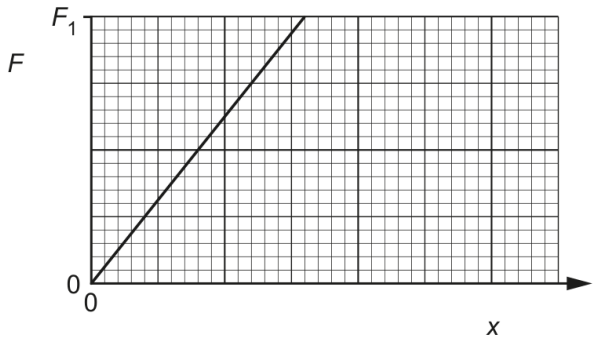


Fig. 4.1

The maximum force applied to the wire is F_1.

The gradient of the graph line in Fig. 4.1 is G. The wire has initial length L and cross-sectional area A.

Determine an expression, in terms of A, G and L, for the Young modulus E of the metal.

E = ... [2]

(iii) Another student repeats the original experiment in **(b)(i)**, increasing the force beyond F_1 to a new maximum force F_2. The new graph obtained is shown in Fig. 4.2.

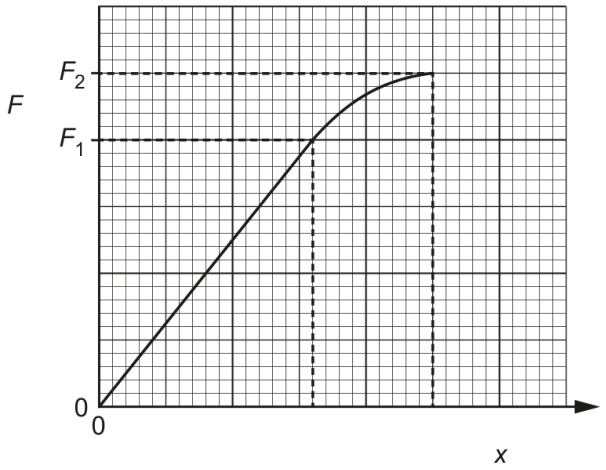


Fig. 4.2

1. On Fig. 4.2, shade an area that represents the work done to extend the wire when the force is increased from F_1 to F_2. [1]

2. Explain how the student can check that the elastic limit of the wire was not exceeded when force F_2 was applied.

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..... [1]

16 (a) A spring is fixed at one end and is compressed by applying a force to the other end. The variation of the force F acting on the spring with its compression x is shown in Fig. 3.1.

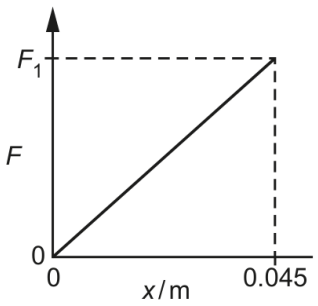


Fig. 3.1

A compression of 0.045 m is produced when a force F_1 acts on the spring. The spring has a spring constant of 800 N m^{-1}.

(ii) Use Fig. 3.1 to show that, for a compression of 0.045 m, the elastic potential energy of the spring is 0.81 J.

[2]

17 The variation with extension x of the force F acting on a spring is shown in Fig. 3.1.

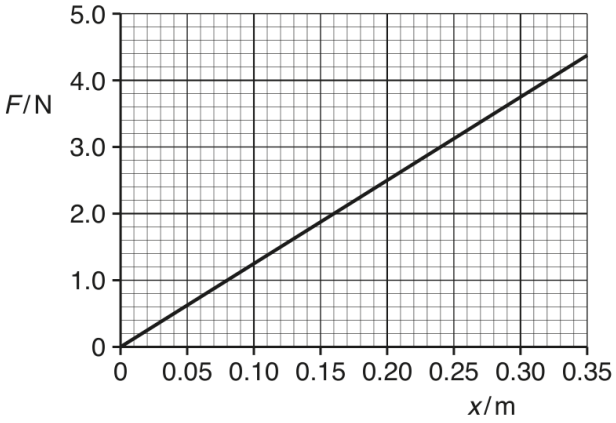


Fig. 3.1

The spring of unstretched length 0.40 m has one end attached to a fixed point, as shown in Fig. 3.2.

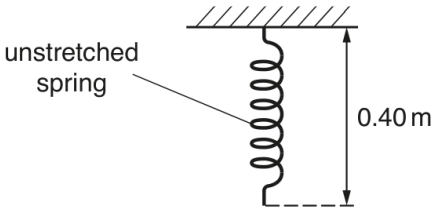


Fig. 3.2

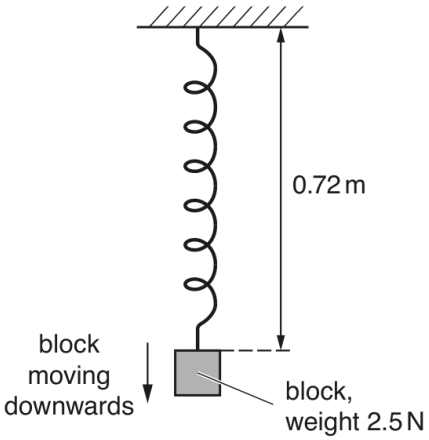


Fig. 3.3

A block of weight 2.5 N is then attached to the spring. The block is then released and begins to move downwards. At one instant, as the block is continuing to move downwards, the spring has a length of 0.72 m, as shown in Fig. 3.3.

Assume that the air resistance and the mass of the spring are both negligible.

(a) For the change in length of the spring from 0.40 m to 0.72 m:

(i) use Fig. 3.1 to show that the increase in elastic potential energy of the spring is 0.64 J

[2]

18 The variation with extension x of the force F applied to a spring is shown in Fig. 4.1.

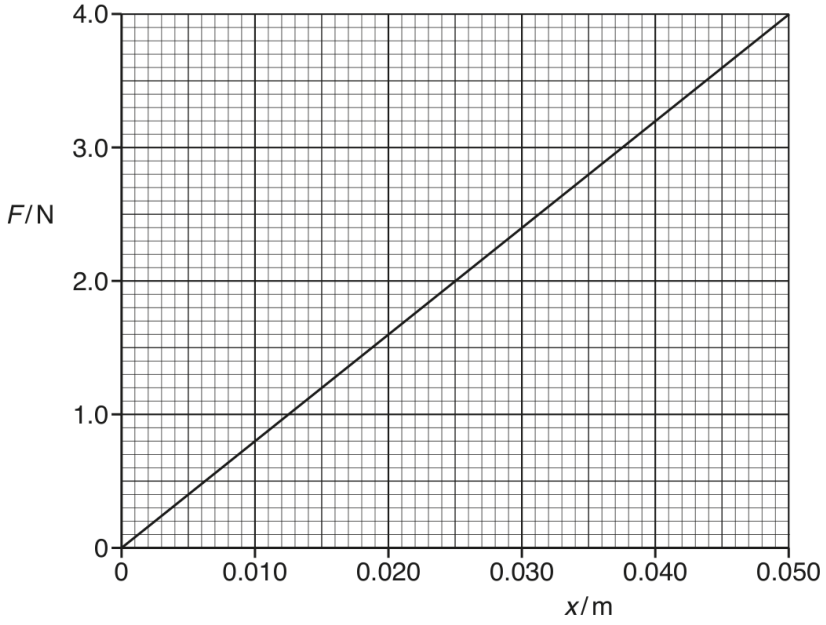


Fig. 4.1

The spring has an unstretched length of 0.080 m and is suspended vertically from a fixed point, as shown in Fig. 4.2.

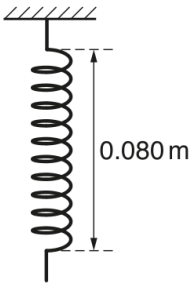


Fig. 4.2

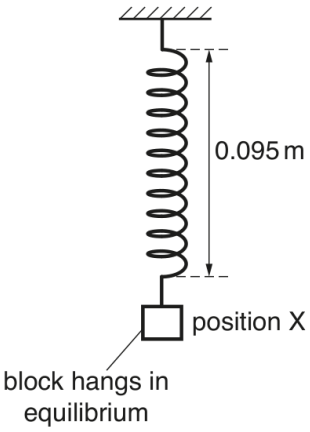


Fig. 4.3

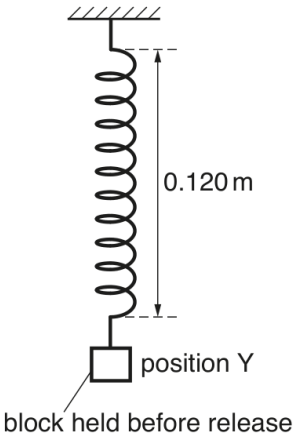


Fig. 4.4

A block is attached to the lower end of the spring. The block hangs in equilibrium at position X when the length of the spring is 0.095 m, as shown in Fig. 4.3.

The block is then pulled vertically downwards and held at position Y so that the length of the spring is 0.120 m, as shown in Fig. 4.4. The block is then released and moves vertically upwards from position Y back towards position X.

(b) Use Fig. 4.1 to show that the decrease in elastic potential energy of the spring is 0.055 J when the block moves from position Y to position X.

[2]